THE LO HADRONIC CONTRIBUTION TO $(g-2)_\mu$

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OUTLINE

- *The EM vs. τ discrepancy for the* $[a_\mu]^{LO}_{had}$
- *Resolving the EM vs. τ puzzle*
- *Some comments on IB (if time permits)*

CURRENT EXPT'L SITUATION (BNL E821)

- $a_\mu \equiv \frac{(g-2)_\mu}{2} = (11\,659\,208.0 \pm 6.0) \times 10^{-10}$ [PRL92 (2004) 161802, $\mu^\pm$ average]

- 0.5 ppm determination (BNL E969 proposal:$\rightarrow$ 0.2 ppm (see previous talk))

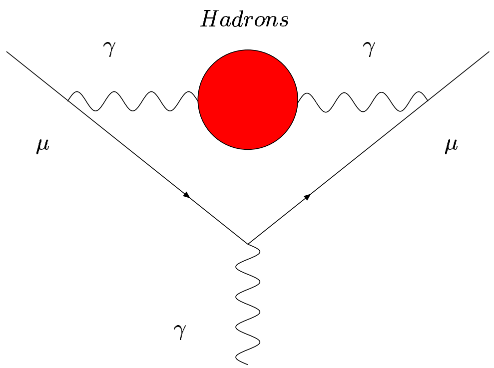
- EM-data-based SM prediction differs from experiment by $2-3\,\sigma$, τ-data-based version does not

SM a_μ SOURCES (M. Passera, hep-ph/0411168)

Source	$\delta(a_\mu) \times 10^{10}$
QED	11658471.88(3)(4)
LO had VP	$\sim 700(6 \rightarrow 8)$
EW	15.4(1)(2)
HO had LBL	13.6(2.5)
HO had VP	$-9.79(9)(3)$

- $[a_\mu]^{LO}_{had}$ largest non-QED, dominant SM error source
- experimental error $< 1\%$ of $[a_\mu]^{LO}_{had}$

The LO Hadronic Contribution



- dispersive representation for $\Pi_{EM}(s)$, $\rho_{EM}(s) = \frac{1}{\pi} Im\, \Pi_{EM}(s)$ via $R(s) = \frac{3s\,\sigma[e^+e^- \to hadrons]}{16\pi\,\alpha_{EM}^2} = 12\pi^2\,\rho_{EM}(s) \Rightarrow$

$$[a_\mu]_{had,LO} = \frac{\alpha_{EM}^2}{3\pi^2} \int_{4m_\pi^2}^{\infty} ds\, \frac{K(s)}{s}\, R(s)$$

with $K(s)$ known

- possibility of using hadronic τ decay input:
  - CVC (isospin) relates flavor ud V current $\rho_{ud;V}(s)$ and $I=1$ EM current $\rho_{EM}^{I=1}(s)$
  - $\rho_{ud;V}(s)$ from invariant mass distribution of flavor ud V current-induced hadronic τ decays ($y_\tau = s/m_\tau^2$, $R_{V;ud} \equiv \frac{\Gamma[\tau^- \to \nu_\tau \, \mathrm{hadrons}_{V;ud}\,(\gamma)]}{\Gamma[\tau^- \to \nu_\tau e^- \bar{\nu}_e(\gamma)]}$)

$$\rho_{ud;V}(s) = \frac{m_\tau^2 \, dR_{V;ud}/ds}{12\pi^2 |V_{ud}|^2 S_{EW} \, (1-y_\tau)^2 \, (1+2y_\tau)}$$

  - ALEPH, OPAL, CLEO (+ B factory) data

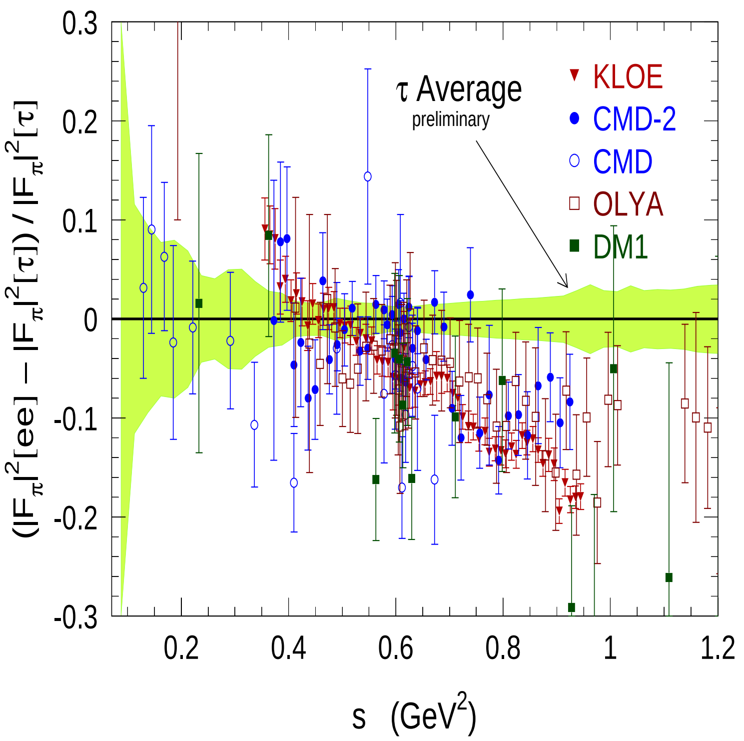
The EM-τ Discrepancy

- $[a_\mu]_{had,LO}$ to $\sim 1\%$ accuracy or better $\Rightarrow$ need IB corrections to τ data/CVC relation

- IB for dominant $\pi\pi$ contribution studied in detail (CEN PLB513 (2001) 361; JHEP 0208 (2002) 002) (*Caution: hep-ph/0509224 for some additional subtleties*), kinematic IB corrections for 4π

- Resulting EM, IB-corrected τ-based $\rho^{I=1}(s)$ *dis*agree
  - $\pi\pi$, $\pi^+\pi^-\pi^0\pi^0$ discrepancies (DEHZ EPJC31 (2003) 503)
  - $\pi\pi$ dominant for $[a_\mu]^{LO}_{had}$

$|F_\pi(s)|$ data, Hocker, hep-ph/0410081



- resulting EM and τ-based $[a_\mu]_{had,LO}$
  - differ by $\sim 2\sigma_{exp} \simeq 2\sigma_{EM}$
  - EM-based a_μ differs from exp't by $2.1 \rightarrow 2.7\sigma$
  - τ-based a_μ differs from exp't by $0.7 \rightarrow 1.3\sigma$
  - KLOE, CMD-2 $[\delta a_\mu]_{\pi\pi}$ compatible within errors $\Rightarrow$ (pre-2005 wisdom) work with EM data only

- *Caution:* point-by-point inconsistencies between CMD2, KLOE data

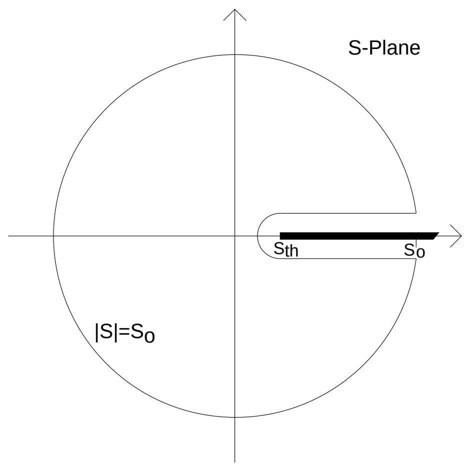
- TANTALYZING SIGNAL FOR POSSIBLE BEYOND-THE-SM PHYSICS, IF SENSIBLE

RESOLVING THE EM-τ DISCREPANCY

- FESR background: $\Pi(s)$, spectral function $\rho(s)$, $w(s)$ analytic $\Rightarrow$

$$\int_0^{s_0} w(s)\,\rho(s)\,ds = -\frac{1}{2\pi}\oint_{|s|=s_0} w(s)\,\Pi(s)\,ds$$

(LHS: data, RHS: OPE [+ $w(s_0) = 0$ to suppress duality violation])



- pFESR OPE features
  - V correlators, $s_0 > 2$ GeV^2: *strongly* $D = 0$ dominated (hence input: $\alpha_s(M_Z)$ from independent high-scale studes, 4-loop running/matching)
  - good convergence of integrated $D = 0$ OPE series
  - "unsuppressed" $D = 2k + 2 \iff c_k \neq 0$, $w(y) = \sum_m c_m y^m$, strong numerical suppression of higher D $O(\alpha_s)$ corrections
  - integrated $D = 2k + 2$ OPE $\sim 1/s_0^k$ (hence test for absence of $D > 6$ contributions)

- THE pFESR ANALYSIS [KM, hep-ph/0504201]

  – FESR choices: various pinched, non-negative, monotonically decreasing $w(y)$, $y = s/s_0$

    * IB-corrected $\rho_\tau(s)$ *uniformly* $> \rho_{EM}^{I=1}(s)$ in region of discrepancy

    * $\Rightarrow$ if τ data correct, (i) EM spectral integrals too small *for all* s_0 (non-negativity), (ii) slope wrt. s_0 too small (monotonicity)

    * $\Rightarrow$ if EM data correct, (i) τ spectral integrals too high *for all* s_0, (ii) slope wrt. s_0 too high

– τ spectral input: ALEPH, CLEO, OPAL, concentrate on ALEPH

– EM spectral input:

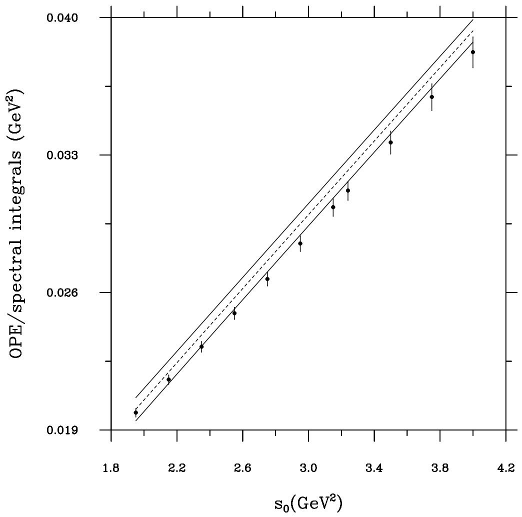
  * large number of experiments, need for VP corrections in many old ones

  * new (2004+) data [KLOE ($\pi\pi$), CMD-2 ($\pi^0\gamma$, $\eta\gamma$, 4π), SND ($4\pi, \omega\pi$), BABAR (3π, 4π)] (only KLOE incorporated into most recent $[a_\mu]_{had,LO}$ determinations)

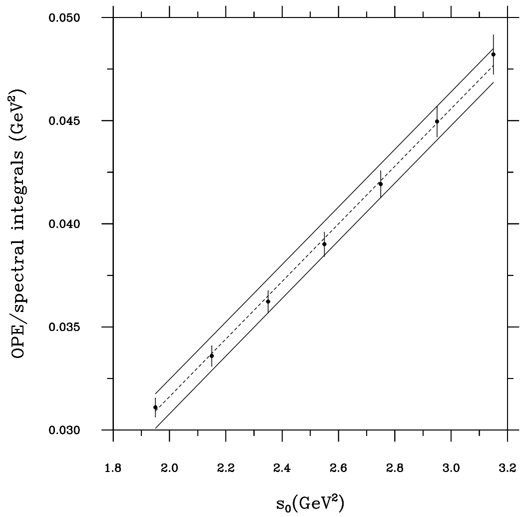
RESULTS

- OPE/spectral integrals for $w(y) = 1 - y$

EM

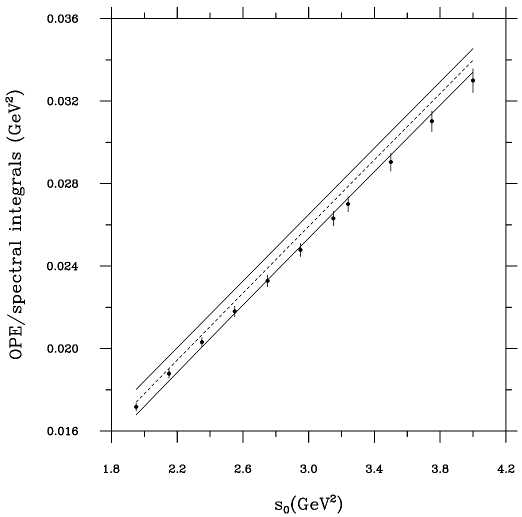


τ

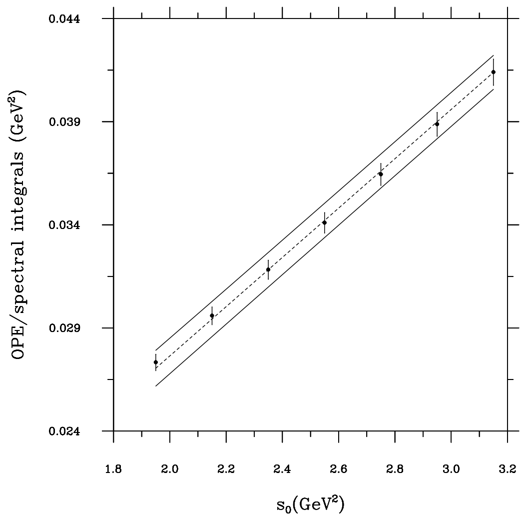


- OPE/spectral integrals for $w_6(y) = 1 - \frac{6y}{5} + \frac{y^6}{5}$

EM



τ



(one of more general "doubly-pinched" weight family, $\{w_N(y)\}$, with $6 \rightarrow N$)

- magnitude, slope of τ spectral, OPE integrals agree for wide range of pinched monotonic ≥ 0 $w(y)$, s_0

- EM spectral integrals $<$ OPE, EM slope $<$ OPE slope for wide range of pinched, monotonic, ≥ 0 $w(y)$, s_0

- EM norm'n problem: $\alpha_s(M_Z)$ to fit $s_0 \sim m_\tau^2$ EM, τ spectral integrals (c.f. high-scale ave: 0.1200 ± 0.0020)

Weight	EM or τ	$\alpha_s(M_Z)$
w_1	EM	$0.1138^{+0.0030}_{-0.0035}$
w_3	EM	$0.1152^{+0.0019}_{-0.0021}$
w_6	EM	$0.1150^{+0.0022}_{-0.0026}$
w_1	τ	$0.1218^{+0.0027}_{-0.0032}$
w_3	τ	$0.1195^{+0.0018}_{-0.0021}$
w_6	τ	$0.1201^{+0.0020}_{-0.0022}$

- EM slope problem: OPE (S_{OPE}) and spectral integral (S_{exp}) slopes with respect to s_0

$w(y)$	S_{exp}	$\alpha_s(M_Z)$	S_{OPE}
$1-y$	$.00872 \pm .00026$	ind	$.00943 \pm .00008$
		fit	$.00934 \pm .00008$
w_6	$.00762 \pm .00017$	ind	$.00811 \pm .00009$
		fit	$.00805 \pm .00009$

"ind": independent high-scale $\alpha_s(M_Z)$ input

"fit": lower input $\alpha_s(M_Z)$ from fit above

CONCLUSION: pFESR results suggest τ data OK, problem with (or non-1γ contributions to) EM data

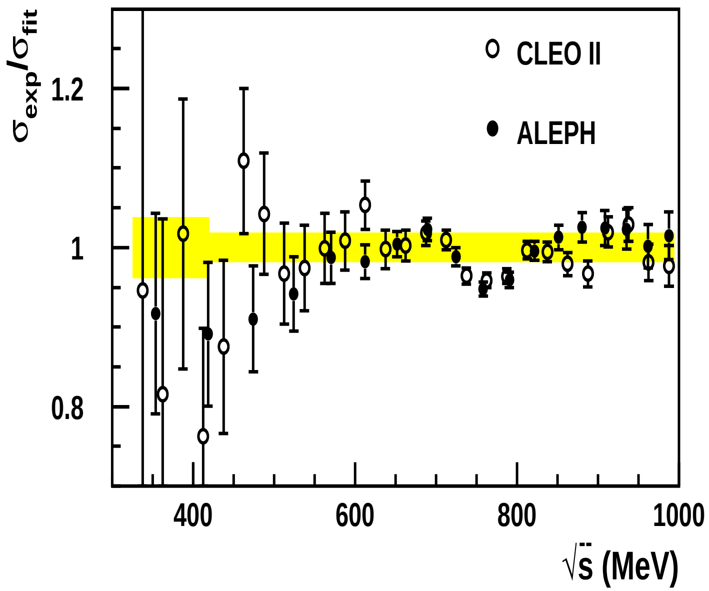
Subsequent (experimental) supporting evidence from SND $\sigma[e^+e^- \to \pi\pi]$ data [hep-ex/0506076]

- SND compatible with IB-corrected τ data in region of previous EM-τ discrepancy [Figure]

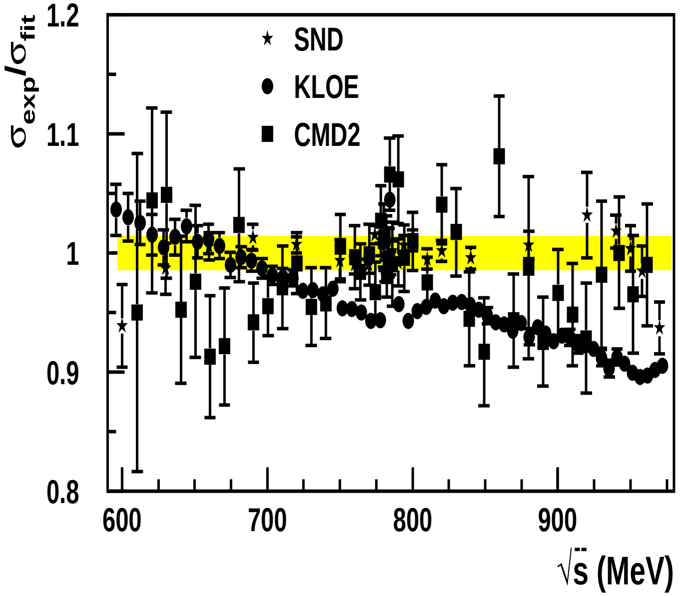
- strong disagreement between SND, KLOE above ρ peak [Figure]

- SND-based $\pi\pi$ $[a_\mu]^{LO}_{had}$ contribution 7×10^{-10} higher than CMD-2 [$(9.1 \pm 6.3) \times 10^{-10}$ c.f. Hocker ICHEP04 average]; compatible within errors with τ-based determination [Davier, Hocker, Zhang hep-ph/0507078]

ALEPH, CLEO τ vs. SND05 $\pi\pi$ data



SND's CMD-2/KLOE/SND05 $\pi\pi$ data comparison



CONCLUSIONS

- pFESR tests, high-scale OPE input clearly favor τ over EM data for V spectral function (supported by SND)

- $\Rightarrow$ τ+EM-based $[a_\mu]_{had,LO}$ favored over determination based on EM data only

- with τ input, SM prediction for a_μ in good agreement with current E821 result

- uncertainties in IB correction needed for use of τ input a limiting feature of this approach, *AND* very unlikely to be reducible below $\sim 4 \times 10^{-10}$ (already larger than proposed BNL E969 accuracy)

- can look forward to new experimental input:
  - BABAR, BELLE radiative return $\pi\pi$ cross-sections
  - Novosibirsk VEPP-2000 upgrade (improved luminosity, $E_{CM}^{max} \rightarrow 2$ GeV (previously 1.38 GeV), upgraded SND, CMD detectors, reduced systematic errors) $\Rightarrow$ significantly reduced errors on exclusive cross-sections (especially useful c.f. current data near threshold, above 1.38 GeV) [scheduled start-up 2005, physics results beginning 2006]
  - BABAR, BELLE hadronic τ decay data with *much* higher statistics than combined LEP experiments

POSTSCRIPT: COMMENTS ON IB CORRECTIONS

- to use τ data for $[a_\mu]_{had,LO}$, need IB corrections

- additional IB uncertainties/corrections not discussed in literature:
  - ρ-ω interference (present for EM, must add for τ):
    - extracted flavor '38' part of ρ_{EM} depends on model for IC and IB amplitudes [Table]
    - model-dependence of integrated contribution to $[a_\mu]_{had,LO} \sim 3\times$ that associated with fitted parameter uncertainties for a given model ($\sim 2\times 10^{-10}$ c.f. standard current CEN estimate 0.8×10^{-10})

Model dependence of $10^{10} \times \left[\delta\,[a_\mu]^{LO}_{had}\right]_{mix}$

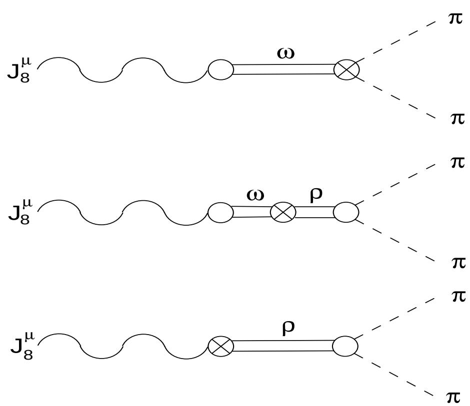
(from fits to CMD-2, SND bare $\sigma[e^+e^- \to \pi\pi]$)

Model	χ^2/dof	$\left[\delta\,[a_\mu]^{LO}_{had}\right]_{mix}$ CMD2 [SND]
GP/CEN	**80.4/42**	$\mathbf{3.5 \pm 0.8}$
GS	35.9/38	2.0 ± 0.5 [2.2]
HLS	36.6/38	4.0 ± 0.6 [4.5]
KS	37.1/38	3.9 ± 0.6 [4.3]
GP/CEN*	40.6/39	2.0 ± 0.5 [1.6]

(NOTE: no acceptable fits of any of these models to KLOE data)

– broad ρ^0-shaped IB contribution (in principle present in EM $\rho^0 \to \pi^+\pi^-$ signal due to IB coupling of isoscalar EM current to ρ^0) not taken into account in present treatments

IB resonance contributions to $e^+e^- \to \pi^+\pi^-$



* $*$ unlike ρ-ω interference, *NOT* experimentally detectable

* $*$ data errors too large to effectively extract using flavor '38' correlator sum rule analysis

* $*$ analogous "direct" IB coupling of π^0 to $A^8_\mu \sim 0.5\%$ of IC coupling at hadronic scales (1-loop ChPT); if similar for vector mesons, would yield integrated contribution $\sim (2-3) \times 10^{-10}$

– current *total* estimated uncertainty on IB correction 2.6×10^{-10} hence significantly underestimated